

Complexity of robust optimization in the polynomial hierarchy

Robust optimization (RO) is a popular framework to handle the uncertainty that arises in optimization problems. The essence of RO lies in imposing that feasible solutions satisfy the robust constraints for all parameter realizations in a given uncertainty set U . We consider in this project the case of min-max combinatorial optimization, which are formally defined as

$$\min_{x \in X} \max_{u \in U} u^\top x, \quad (\text{ROBUST})$$

where $X = \{x \in \{0, 1\}^n \mid Ax \geq b\}$ and $U = \{x \in \mathbb{Z}^n \mid Cx \leq d\}$ are both discrete sets. Despite being both discrete sets, observe that X and U play different roles in (ROBUST). On the one hand, X models the structure of the underlying combinatorial problem at hand. For instance, X could be the feasibility set of the knapsack problem or shortest path problem. On the other hand, U models the sets of scenarios for the cost vector.

In what follow, we consider the decision counterpart of (ROBUST):

$$\min_{x \in X} \max_{u \in U} u^\top x \leq K? \quad (\text{DEC})$$

Problem (DEC) has been introduced decades ago and its complexity is well-understood whenever set U is a polytope, rather than a discrete set. In this case, the problems happens to be NP-complete in the strong sense in general. However, the introduction of integrality restrictions to U leads to problems that are higher in the polynomial hierarchy. Specifically, it has been shown in [1] that problem (DEC) is Σ_2^P -complete. Yet, the result of [1] leverages arbitrary structures for X and U and does lead to insights for specific sets X and U .

Observe that swapping the order of the min and max leads to a different problem since in this case the decision maker can choose x knowing already the value taken by the uncertain parameters c . The resulting problem is called an *interdiction* problem

$$\max_{u \in U} \min_{x \in X} u^\top x, \quad (\text{INTERDICTION})$$

Unlike the general result of [1] based on arbitrary sets X and U , the authors of [2] provide much more precise results and leverage the structure of X to show, in essence, that whenever X models a NP-complete problem, the decision version of (INTERDICTION) is Σ_2^P -complete.

Objective The purpose of this internship is to dig into the results of [2] to come up with similar classification for (DEC).

Context The internship will take place at the LIRMM, in Montpellier. The student will be advised by Michaël Poss. This internship might lead to a PhD related to more difficult complexity questions in robust adjustable optimization.

References

- [1] Matthias Claus and Maximilian Simmoteit. A note on $\sigma 2p$ -completeness of a robust binary linear program with binary uncertainty set. *Operations Research Letters*, 48(5):594–598, 2020.
- [2] Christoph Grüne and Lasse Wulf. Completeness in the polynomial hierarchy for many natural problems in bilevel and robust optimization. In *International Conference on Integer Programming and Combinatorial Optimization*, pages 256–269. Springer, 2025.